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ORIGINAL ARTICLE

The role of 2D and 3D mental rotation in mathematics for young children: what is it? Why does it matter? And what can we do about it?

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Abstract The ability to mentally rotate objects in space has been singled out by cognitive scientists as a central metric of spatial reasoning (see Jansen, Schmelter, Quaiser-Pohl, Neuburger, & Heil, 2013; Shepard & Metzler, 1971 for example). However, this is a particularly undeveloped area of current mathematics curricula, especially in North America. In this article we discuss what we mean by mental rotation, why it is important, and how it can be developed with young children in classrooms. We feature results from one team of teacher-researchers in Canada engaged in Lesson Study to develop enhanced theoretical understandings as well as practical applications in a geometry program that incorporates 2D and 3D mental rotations. Children in the Lesson Study classrooms (ages 4–8 years) demonstrated large gains in their mental rotation skills during 4 months of Lesson Study intervention in the Math for Young Children research program. The results of this study suggest that young children from a wide range of ability levels can engage in, and benefit from, classroom-based mental rotation activities. The study contributes to bridging a gap between cognitive science and mathematics education literature.

Keywords Spatial reasoning · Mental rotation · Mathematics · Young children · Geometry

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1 Overview of spatial reasoning

Geometry in elementary schools typically involves activities of naming shapes, determining their properties, sorting and classifying them (Copley, 2000). How many sides does a triangle have? What is the sum of the angles of a square? How many faces are there on a cube? Are cubes and rectangular prisms in the same family? Usually the configuration of these 2D and 3D figures in school mathematics are fixed and unchangeable. This simplified and static conception of geometry is reflected in current mathematics programs in Canada and the US—where objects are rarely moved, transformed or re-shaped, and figures in the environment are rarely visualized from different perspectives. Although neglected, dynamic transformational geometry—thinking about how shapes move, change, interact in space, and how we move in relation to shapes and figures—is an important construct of geometry, namely that of spatial reasoning. According to Kinach (2012),

> Spatial thinking takes a variety of forms, including building and manipulating two- and-three-dimensional objects; perceiving an object from different perspectives; and using diagrams, drawings, graphs, models, and other concrete means to explore, investigate, and understand abstract concepts such as algebraic formulas or models of the physical world. (p. 535)

Kinach goes on to explain the central role spatial reasoning plays in mathematics overall:

> Whereas geometry is the obvious example of spatial reasoning in mathematics, the mathematician Jacques Hadamard argues that "much of the thinking that is required in higher mathematics is spatial in nature"

> (Jones 2001, p. 55). Indeed, spatial ability is known to be a predictor of mathematics achievement at all grade levels (Clements 1992). (p. 535)

Cognitive scientists, Farmer et al. (2013) concur: their research demonstrates that spatial reasoning plays an important role in predicting overall mathematics success with even greater predictive power than general mathematics scores. Spatial reasoning is also a feature of our day-to-day lives of navigating space and is essential for careers in Science, Technology, Engineering, the Arts, and Mathematics (Newcombe & Frick, 2010; Wai, Lubinksi, & Benbow, 2009).

Spatial reasoning has been particularly well studied in psychology, but less so in mathematics education. Nonetheless, spatial reasoning has received attention by some mathematics education organizations such as the National Council for Teachers of Mathematics in the United States, who recommended in 2008 that at least half of the early mathematics curriculum be focused on geometry, measurement, and spatial reasoning; however, resources for doing so are limited in quality and/or not widely in use (Clements & Sarama, 2004, 2011; Ehrlich, Levine, & Goldin-Meadow, 2006). This is confirmed in Ontario Canada, where a recent survey ($n = 620$) revealed that Kindergarten to Grade 2 classroom teachers devoted the least amount of time to geometry and spatial sense compared to the other mathematics strands (Bruce, Moss & Flynn, 2012). To add further complexity, young children who are disadvantaged (particularly due to low SES circumstances, as defined by total family income and mother's education) perform poorly on spatial reasoning tasks (Farmer, et al., 2013) compared to their higher SES peers. Without intervention these gaps will likely not close (Jordan & Levine, 2009). Unfortunately, intervention studies are relatively scarce compared to the volume of studies that document the problems of classroom practice without offering solutions (Stylianides & Stylianides, 2013).

2 The importance of mental rotation—what does this mean and why does it matter?

One type of spatial reasoning is the skill of performing mental rotations. Performing mental rotations involves moving 2D or 3D objects around one or more axes in the mind's eye (Shepard & Metzler, 1971), and it is often measured by having people identify images of matching shapes that are presented in different orientations or are decomposed and rotated (see Fig. 1).

Although mental rotation is a feature of geometry in and of itself, it is important to recognize that mental rotation is also a cognitive process studied by cognitive scientists and neuroscientists. Developing mental rotation skills provides children with a 'cognitive tool' that can be wielded throughout school mathematics. For example, mental rotation can be used as a reliable strategy for understanding area measurement tasks (see Fig. 2), composing and decomposing 2D and 3D figures, proving symmetry, and finding missing addends in number (Cheng & Mix, 2013). And yet, we tend to treat mental rotation as a skill that is divorced from geometry and all other areas of mathematics. Some argue on the other hand, that just as arithmetic is foundational to a wide assortment of mathematical tasks, so too is the ability to perform mental transformations of objects and shapes (Jones, 2001).

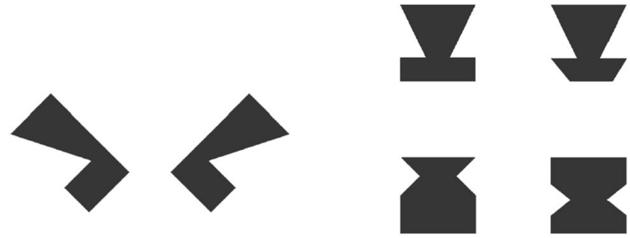


Fig. 1 When we mentally rotate the *two shapes on the left* so that they are joined at a centre *y* axis, which figure do they make (of the *four on the right*)? (From Levine CMTT, et al., 1999); see also the classic test of Shepard & Metzler, 1971

Fig. 2 When asking children to think about the area of these two squares, students describe mentally rotating the left square to match the square on the right as a proof that the area of the two squares are the same

Similarly, 3D mental rotations typically involve identifying two equivalent figures where one is a target figure, and the participant must identify which figure is an exact match once it has been rotated. As with the 2D mental rotation task, one of the choices is an exact match while the others are not (see Fig. 3).

Mental rotation skills are closely linked to skills such as map reading (Pazzaglia & Moè, 2013), orientating and navigating (Linn & Peterson, 1985), verbal and visual-spatial working memory (Kaufman, 2007) and also to overall problem solving (Geary, Saults, Liu, & Hoard, 2000). These skills are used in everyday living but 3D mental rotation abilities are also clearly linked to mathematics in the curriculum including school geometry (Delgado & Prieto, 2004), algebra (Tolar, Lederberg, & Fletcher, 2009), and mental mathematics (Kyttälä & Lehto, 2008).

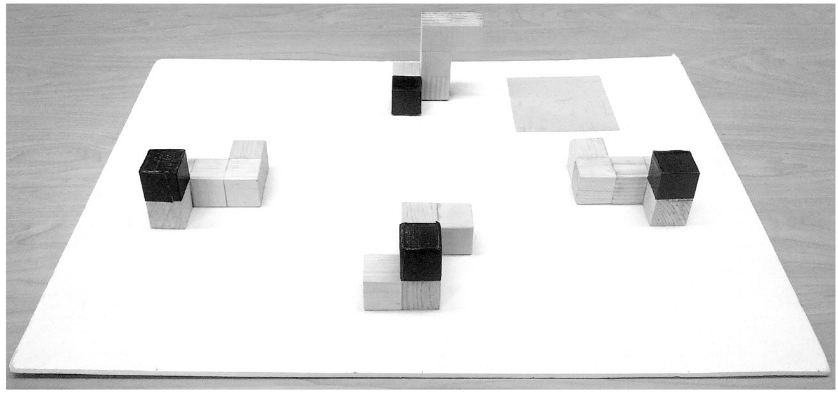


Fig. 3 In this 3D mental rotation blocks task (3DMRBT: Hawes, LeFevre, Chang & Bruce, 2014), the participant must identify which of the three figures at the front exactly matches with the figure at the back once rotated

There is little research on 3D mental rotation with young children, and what is currently available lacks clarity: a number of recent studies have found that 3D mental rotations are too challenging for elementary school children (Hoyek et al., 2012; Jansen, Schmelter, Quaiser-Pohl, Neuburger, & Heil, 2013). Jansen and colleagues (2013) tested children in 2nd and 4th grade ($n = 449$) using three types of mental rotation tasks (3D animal drawings, 2D letters, or 3D cube figures). The cube figures were similar to those used in mental rotation tasks designed for adults where the figures are images—not real objects. Children demonstrated good performance with the first two types of tasks but children performed only at chance with the 3D cubes task (see also Hoyek et al., 2012). On the other hand, some studies are finding that engaging with cubes as real objects may be within reach for young children. For example, Örnkloo and von Hofsten 2007 demonstrated that by the age of 22 months, young children are cable of physically rotating variously shaped 3D objects to fit into matching apertures; a feat said to rely on mental rotation. In recognition that many of the traditional measures of mental rotation are inappropriate (or not well suited) for young children, Casey and colleagues 2008, designed their own task-based measure using multi-link cubes. Four- and five-year-old children were presented with two matching figures composed of multi-link cubes. The researcher then covered the matching figures and changed the orientation of one of the figures. The child was then given a maximum of 10 s to re-align the altered figure with the target figure. While all children were able to perform this task, there were individual differences in the length of time required to complete the task. Moore and Johnson have shown (2008 and 2011) that infants can distinguish between 3D cube figures and their rotated mirror figures even though this skill poses challenges to many adolescents and adults (Cooper, 1992; Ozdemir, 2009). In our recent research with children ages 4 through 8 (Bruce, Flynn & Moss, 2013; Bruce & Flynn, 2012) we have found that 5-year-olds performed above chance on items such as the one displayed in Fig. 2 (Hawes et al., 2014) where real cubes were used rather than images of cube figures.

Just as general spatial reasoning is trainable, there is now sufficient evidence that spatial reasoning—including mental rotation—is malleable (Feng, Spence & Pratt, 2007; Uttal et al., 2013). In their recent meta-analysis Uttal et al. (2013) found that spatial thinking can be improved through targeted training across all ages through a wide assortment of interventions. For example, Terlecki and colleagues 2008 demonstrated that it is possible to improve mental rotation through two different conditions: repeated testing and playing the videogame Tetris. Interestingly, while the undergraduates in both conditions improved in their mental rotation performance, only those in the videogame condition demonstrated improvements in several other spatial tasks. The associated gains in mental rotation skills were still present several months later.

3 Can mental rotation skills be improved through practice and classroom instruction?

Despite the claim that "the relation between spatial ability and mathematics is so well established that it no longer makes sense to ask whether they are related" (Mix & Cheng, 2012, p. 206), the majority of research in this area remains divorced from pedagogical practice and the 'every day' realities of the classroom. Determining whether it is possible to improve mental rotation performance through educational interventions and training is of both theoretical and practical importance. The research linking spatial thinking to overall mathematics performance provides a promising signal to pursue research into classroom implications: We can hypothesize that if we provide learning environments that foster the development of students' mental rotation skills, then we might also see associated gains in their overall mathematics performance. The strength of this hypothesis lies in a conviction that mental rotation operates not only as a specific and isolated skill, but also as a fundamental skill that permeates the discipline of mathematics as a whole.

One area in need of concerted research efforts alluded to earlier are studies designed to demonstrate the causal relationship between spatial learning and mathematics. To our knowledge, only one such study exists (despite over a century of knowledge on the link between space and mathematics). Cheng and Mix (2013) randomly assigned children to either a spatial training condition (i.e., mental rotation practice) or crossword puzzle condition. Both groups completed the same pre and post-tests, assessing both spatial reasoning and mathematics skills. Children in the spatial training group, but not the crossword condition, demonstrated significant improvements in their calculation skills, especially on missing term problems (e.g., $4 + \_\_ = 10$). While experimental research of this sort is essential to elucidate the causal relationship between spatial thinking and mathematics, research is also needed that bridges the gap between 'lab-based' research and classroom practice. This is the second area we have identified as requiring more research attention and acts as the motivation behind the study presented in this paper.

Our central research question was, "What impact does an in-class intervention have on students' mental rotation and spatial thinking?" Motivated by this question, we designed a study with the intention of achieving two interdependent research objectives: (1) to work with early years teachers to co-design and implement activities and lessons that emphasize a 'spatial approach' to mathematics learning, aimed at supporting young children's spatial thinking and mental rotation skills, and (2) to determine whether such an approach has an effect on students' spatial thinking, namely mental rotation skills.

4 A recent development of school-based intervention

As described by Stylianides and Stylianides (2013)

> research on classroom-based interventions in mathematics education has two core aims: (a) to improve classroom practice by engineering ways to act upon problems of practice; and (b) to deepen theoretical understanding of classroom phenomena that relate to these problems. (p. 333)

Three key features of mathematics education research on interventions outlined by these same mathematics education researchers are as follows: (1) the research is conducted in classrooms through collaboration of teachers and researchers to ensure relevance and practicality; (2) the research explicitly addresses areas of mathematics that are difficult to learn and difficult to teach; and, (3) the research is empirically tested to show not only that the interventions have a positive impact on student learning but also explains or illustrates why they are effective (Stylianides & Stylianides, 2013). In keeping with these criteria, our Ontario Canada mathematics research team has been developing a 'no-ceiling' approach to working with children from ages four through seven and their teachers, in a research program called Math for Young Children (see http://tmerc.ca/m4yc/). This work includes the use of Lesson Study as a mathematics professional learning model (Bruce & Ladky, 2011; Bruce, Flynn, Ross & Moss, 2011). Lesson Study involves a series of four key steps repeated in cycles: (1) curriculum and lesson goal setting; (2) planning of tasks and lessons that match the goals and the needs of students; (3) implementation of the tasks and lessons with students in the classroom; and (4) debriefing the functioning, benefits and challenges of the tasks and lessons implemented. At this stage, the team usually refines the goals and tasks for future implementation.

In our Math for Young Children research program, researchers from the team have worked with over ten teacher teams to investigate ways to spatialize the mathematics curriculum in their classrooms, with a particular focus on the strands of geometry and measurement. Through the Lesson Study process, teams have engaged in task design both in the form of pre-post clinical interview activities for individual students and in the form of classroom lessons (Bruce, Flynn & Moss, 2013; Bruce & Flynn, 2012). These tasks were then field-tested in classrooms, over time and across teams. Throughout the Lesson Study professional learning program, teachers posed research questions, which changed over time reflecting the evolution of the teams' interests (Sakonidis & Potari, 2014) and growing understanding of what it means to reason spatially. School A, the site featured in this article, documented the evolution of their inquiry questions through out the Lesson Study research process (see Fig. 4).

4.1 Participants at School A

Due to the complexity and volume of data for the project, the descriptive findings from one team of 7 teachers and 42 of their students are featured here, namely those of School A. School A is an urban Kindergarten to Grade 8 school in a mid-sized city in Ontario. The students in this low socioeconomic status community come from a mix of cultures with English being the predominant language in the home. The community is relatively underprivileged with strong need for breakfast and lunch food supplement programs. There is also a high degree of need for specialized supports due to the range of learning challenges the students face. The students of this school are lagging behind the mean (by 12 %) of the province for mathematics achievement (based on annual Education Quality and Accountability Office provincial testing results, see http://eqao.ca).

Fig. 4 Evolution of the research questions for the team of teachers at School A as documented during meetings of the team

Team Research Questions Documented Over Time
Initial Questions: • What are the connections between geometry and spatial sense in the students' world? • Do students know the language (e.g., attributes)? • Are the students able to compose and decompose irregular shapes? Revised Questions (evolution 1): • Does physical practice improve mental ability to compose/decompose and perform mental rotations? (Levine) • How does gesture help? (Do gestures fall off as students get older?) Revised Questions (evolution 2): • How do students understand transformations (rotations and reflections in particular)? • Are students doing enough composing and decomposing of shape? • What are our own gestures and how are they affecting student thinking and student gestures? Revised Questions (evolution 3): • How do we move students from 2D thinking to 3D thinking (rotations, orientation & location)? • How do students interpret photos of 3D figures compared to diagrams of 3D figures? • How can we prompt students to use more gestures in their mathematics, especially when performing mental rotations?

5 Method

5.1 Data collection

While all students participated in the mathematics intervention, each teacher selected six students from their classrooms to participate in the pre- and post-test assessments. A total of 42 students aged 4–8 years of age participated, some from each grade ranging from Junior Kindergarten to Grade 2 (mean 6.3 years, SD 1.2, range 4.2–7.9). Teachers selected students based on our request to sample a range of student ability levels in mathematics. That is, we asked the teachers to select students that reflect what they considered to be low-, middle-, and high-performing mathematics students. In total, 17 low-, 14 mid-, and 11 high-performing students participated. Each participating student ($n = 42$) was interviewed individually by a trained research assistant in a quiet location at the school. At both time points (pre and post intervention), students were assessed on an identical battery of spatial reasoning and mathematics assessments. In this paper, we report only how students performed on the two spatial reasoning measures employed: 2D mental rotation and 3D mental rotation. In order to control for potential priming effects, children completed the two measures on separate days, completing the 2D measure first and the 3D measure second. Of the 42 participants, 38 completed the pre and post assessments of 2D mental rotation, while 39 completed the 3D mental rotation assessment at both time points. We were not able to collect pre-post data on all 42 individuals due to absenteeism.

All interviews were video recorded for three purposes: First, the video-recorded task-based interviews were used to verify the accuracy of the research assistants' in-person scoring. Second, video excerpts were used in professional learning sessions and presentations with educators and policy makers. These video examples were used not only to illustrate what students were expected to do in the interviews, but also to demonstrate the abilities of young children to reason spatially, particularly when the adults in these professional learning sessions were finding the tasks challenging themselves. Third, the interview video data are currently being used for additional analysis of gestures used by the children during the 2D and 3D mental rotation tasks. Gesture analysis is still underway and not reported in this paper.

5.2 Student measures

5.2.1 2D mental rotation task

The 2D mental rotation task was adapted from the Children's Mental Transformation Task (CMTT, Form D; see Levine et al., 1999), a widely used measure of young children's (4- to 7-year-olds) spatial ability in psychological research (see Levine et al., 1999, Harris, Newcombe, & Hirsh-Pasek, 2013; Hawes et al., 2014). The original 32-item task consists of questions dealing with both translations and rotations. However, we adapted the task to only include rotational items (16 total test items). Participants were asked to identify which of four figures would result from mentally rotating and joining two congruent mirrored shapes (see sample in Fig. 1). Participants were awarded one point for each correct response; thus, the maximum score on this task was 16. As previously noted, this ability to mentally rotate objects in space is a key metric of spatial

Table 1 Video data summary from School A

Type	# of episodes	Amount of video in time
Clinical interviews	38 × 2 (pre/post)	13 h 55 min 50 s
Meetings	2	4 h 8 min 13 s
Lessons	6	3 h 1 min 17 s
Total video set 1	87	21 h 5 min 20 s

reasoning (Frick, Ferrara, & Newcombe, 2013; Levine et al., 1999). Some recent evidence even suggests that spatial reasoning measures might even be better predictors of later mathematics performance than number-based assessments (see Verdine et al., 2014 and Farmer et al., 2013).

5.2.2 3D mental rotation block task

The 3D mental rotation block task required students to examine one 3D block figure and match it to one of three other 3D block figures: one was an exact match but rotated, one was a mirror figure, and the third was a distractor figure (see Fig. 3). This task also consisted of 16 test items. Children were awarded one point for each correctly identified match.

5.3 Additional data sources

Qualitative data were collected on students and teachers through a variety of means. Students and teachers were video recorded during the various lessons, individual interviews, and activities that were carried out. Trained research assistants also conducted detailed field notes during the PD meetings as well as during in-class activities and lessons. An end-of-year audio-recorded focus group interview was also used to collect data on teacher participants during the last meeting. In total we collected and analyzed over 21 h of video footage. Table 1 shows the types of activity that were video recorded, number of video episodes, and the amount of time recorded for School A in the Math for Young Children project.

In addition to the three purposes of video collection of student interviews outlined in Sect. 5.1, the video of lessons combined with interview video became a valuable source of information for teacher participants. By viewing, stopping, and re-viewing video of their students in interview and classroom contexts, the participating teachers were able to closely examine student thinking—something they reported was difficult to do in their busy classrooms. This led to detailed reflection on teacher moves in the classroom and on what types of tasks needed to be implemented in the classroom.

5.4 A classroom intervention focused on mental rotation and spatial reasoning

The teacher-researcher team at School A met over seven full days punctuated throughout a 4-month period. At these meetings, researchers provided up-to-date research information on spatial reasoning in the form of carefully selected articles related to expressed interest of the team along with brief oral summaries of these articles. The researchers also introduced four geometry tasks for teachers to try at some meetings to incite discussion about the mathematics content of the tasks. A series of Tangram challenges, for example, was introduced during the second meeting of the team. During the meetings the teachers and researchers also co-developed a series of approximately 14 tasks (at least two at each meeting). Also, at four of these meetings, voluntary teachers implemented the tasks in one of the classrooms while the rest of the team observed students in that same classroom. This was followed by a discussion and a revising of these tasks for use in all of the participating classrooms. At each session, every classroom teacher debriefed the implementation of several common tasks (and teacher variations to these tasks) using field notes, photos, and/or video that teachers and/or researchers had collected in order to share evidence of student thinking, successes, and challenges. By the end of the 4 months, all tasks were field-tested in the participating classrooms with the central goal of increasing spatial reasoning experiences and skills of students—with particular attention to 2D and 3D mental rotation. For example, the teachers designed and tested a lesson called “The Four Cube Challenge” where students connected four interlocking cubes to generate a series of structures. The teachers explicitly drew attention to configurations of four interlocking cubes that were unique, having students rotate their figures first mentally and then physically, to check for congruence/equivalence with other four-cube figures. In this task, when a child is building a four-cube structure with interlocking cubes such as the one displayed in Fig. 5, (s)he is (1) selecting and rotating individual cubes and joining these together to make one structure; (2) examining the generated figure and rotating it in space to lay it down on a horizontal surface; (3) comparing this figure to others for their properties to assess whether they are similar, equivalent or different; and finally, (4) using mental rotations and then physical rotations to compare the four-cube structure to other four-cube structures.

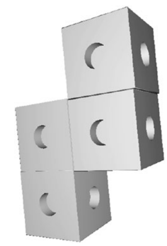


Fig. 5 Example four-cube structure

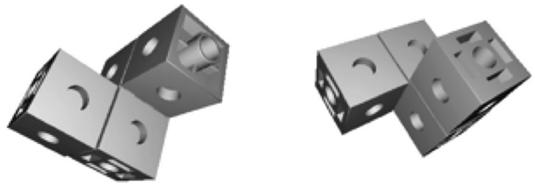


Fig. 6 Two four-cube structures that are mirror figures (embedded in an iBook format for students to use after their explorations with interlocking cubes)

Performing the mental rotations necessary to compare figures can be particularly challenging with mirror figures such as those presented in Fig. 6.

In our work (see Bruce, Flynn & Moss, 2013), we have seen both students and their teachers challenged by the task of identifying, differentiating, and describing mirror images.

Researchers and teachers documented all of these co-developed tasks that the teachers introduced to their students. Table 2 shows the general progression of the mathematics tasks that the teachers and researchers co-developed and that the teachers implemented in their classrooms at School A. The table illustrates how the team moved from observing play contexts where mathematics was likely to be found, to composing and decomposing 2D and 3D figures, to mentally rotating 2D and 3D figures and verifying congruency in a concrete rotation and matching strategy, to practicing performing mental rotations, to building 3D figures by rotating and fitting cube formations together based on 2D diagrams and photos. In each phase, the topic of mental rotation was approached and discussed by the team. As the team's key mathematics goals narrowed in focus, so too did their ability to notice and analyze children's mental rotation strategies across the various tasks. For example, in discussing children's block play at the beginning of the study, the topic of mental rotation was alluded to, but never explicitly addressed. By the time the team was planning for their cumulating lesson, however, the team had very specific goals with regards to the improvement of children's mental rotation skills. Over time—and through many group discussions—the team became fluent in "mental rotation." Team members were then able to observe, identify, and explicitly address children's use of mental rotation strategies during block play.

5.5 Data evaluation

The research team examined results from the classroom interventions both qualitatively and quantitatively (descriptive data) in order to gain a more comprehensive understanding of the impact of the interventions on student learning and a clearer image of what effective classroom interventions might "look like" (Stylianides & Stylianides, 2013). For the quantitative measures, the first level of analysis involved isolating the 2D and 3D results to examine pre and post means, paired-samples t test results, p values and effect sizes for each of these two measures. We then combined the 2D and 3D results in a second level of analysis, to look for overall trends and significance in the results. For the qualitative measures, researchers analyzed field notes of the teacher meetings and transcript from the focus group interview using a full thought utterance as the unit of analysis. The field notes and transcript were analyzed using two stages of coding. The first stage involved open coding, where each utterance from the text was highlighted and named using key words from the utterance. These codes were then listed and clustered into two broad categories: (a) opportunity to learn and (b) shifts in teacher estimations of student abilities. Specific methods of analysis are more fully described in the findings sections that follow.

6 Findings

6.1 2D mental rotation test results pre and post

Table 3 illustrates the pre-post results for the 2D mental rotation task. Although the sample is small, students in all grade levels made large gains in their 2D mental rotation performance. Paired-samples t tests revealed that with exception of the lowest age group (Junior Kindergarten students, ages 4–5) all other grade levels demonstrated significant gains in their 2D mental rotation performance ($p \leq 0.001$). The magnitude of these effects can be seen by comparing post-test scores to pre-test scores across the different grade levels. For example, students in Senior Kindergarten (ages 5–6) achieved a mean score of 9.25 at post-test, which surpasses the mean pre-test scores of the Grade 1 students (ages 6–7) and nearly equals the mean score of the Grade 2 (ages 7–8) students at pre-test, 9.92.

6.2 3D mental rotation block task pre and post results

Table 4 illustrates the pre-post results for the 3D mental rotation block task. Improvements were demonstrated by all age groups; however, the effects were smaller and less consistent than those observed on the 2D task. Paired sample t tests revealed that children in Junior Kindergarten (ages 4–5) and Grade 1 (ages 6–7) did not demonstrate significant gains ($p > 0.05$). Significant gains were demonstrated, however, by children in Senior Kindergarten (ages 5–6) and Grade 2 (ages 7–8; $p < 0.05$). It is not surprising that the children in Junior Kindergarten did not make significant gains, given that they also failed to make significant gains on the 2D mental rotation measure. Previous

Table 2 Teacher-researcher mathematics goals and task descriptions

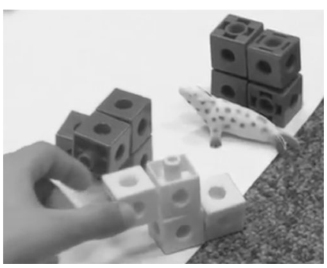
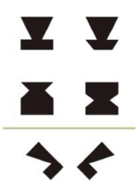
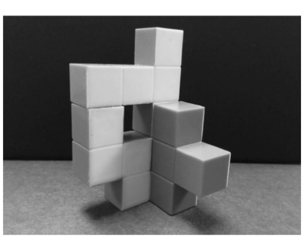
Key mathematics goals	Tasks descriptions
Block play observations Observe students in play with mathematics blocks to consider how as teachers we can mathematize children's play 	(a) Using a detailed observation guide, teachers observed students at free play with wooden mathematics blocks. Key points of observation included: use of mathematics language including positional language, orientation, symmetry, complexity of the design, and persistence. (b) Teachers then set-up and observed a structured play problem using the same observation guide: Problem 1: How many different (non-equivalent) figures can you make with these cubes? Problem 2: Create a figure from diagrams of the top, front, and side views. The students rotated blocks physically as part of their problem solving through play, while teachers made observations
Pattern blocks and tangrams challenges Composing and decomposing 2D figures in a playful context of a series of challenges to encourage the language and noticing of position, orientation 	Using pattern blocks and/or tangrams, students were given a series of challenges (e.g., make a large hexagon using many pattern blocks; make a large triangle using two triangles from the tangram set). Learning was consolidated with a 'gallery walk' (a walk about the room as though in an art gallery) to look at the designs others made and then discussion of 'What did we discover?' In this task, students were required to physically rotate 2D shapes to compose larger shapes, providing students with many opportunities to manipulate and rotate figures kinesthetically
"Uniqua"—exploring 2D and 3D rotations Determining congruence of 2D shapes and equivalence of 3D structures by performing mental rotations and then verifying by rotating and comparing the figures 	(a) Students were presented with congruent but rotated 2D shapes (made of paper) and asked to decide if the shapes were congruent or not. And then without touching the shapes describe what they would have to do to make them look the same (mental rotations). The students then verified their mental rotations with the paper shapes (overlaying the shapes after rotating) (b) Students were then asked to analyse 3D figures made from five cubes. They had to determine if any of the figures were equivalent (the same) by comparing them first without moving them. Students were asked what they would have to do to the figure to make them the same. Then they verified for congruence by rotating one figure to match another (c) When presented with one 3D "house" for the character Uniqua, students were asked to make a non-equivalent or 'unique' house for Uniqua using the same number of cubes. Students then compared their houses to predict whether there were any that were equivalent (the same)—this involved students having to mentally rotate the structures to compare them in their mind's eye and then compare them physically
Inverse Levine (CMTT) Mental rotations combined with decomposing 2D shapes 	A booklet of images was created (based on Levine's CMTT task for clinical interviews). The top images showed four different composed shapes; the bottom image showed one of the four shapes partitioned in half AND rotated Prompt: If you could cut these four shapes in half (point to the four shapes), which one would give you these two pieces (point to the two pieces)? This task provided students with opportunities to practice decomposition and mental rotations of 2D figures
Exploring 3D pentominoes Linking 2D images to the construction of 3D structures 	The final tasks consisted of three phases of exploration with 3D pentominoes blocks. Phase 1 encouraged exploratory play with the pentominoes (in pairs). Phase 2 focused on making structures from photos. In Phase 3, students built a structure from a drawing (instead of a photograph). Tasks were increasingly difficult: at the beginning students were shown the component pieces and the structures were relatively simple; at the end they were told only the number of pieces needed and the structures were more complex. Students were required to rotate the 3D pentominoes figures to generate the structures in the images, but were also asked to predict what they would have to do to the figures to make them 'match' the structure in the image. This led to increased gesturing and spatial language

Table 3 Pre-post improvements on the 2D mental rotation task (maximum score 16)

Grade	Mean age (years)	Mean pre-test (SD)	Mean post-test (SD)	*T*	*P*	Effect size
JK ($n = 8$)	4.5	4.25 (1.39)	6.38 (3.50)	1.92	0.097	0.77
SK ($n = 8$)	5.7	6.63 (3.42)	9.25 (3.65)	5.70	0.001	0.74
1 ($n = 10$)	6.8	7.40 (3.60)	11.50 (2.95)	7.24	<0.001	1.30
2 ($n = 12$)	7.5	9.92 (2.68)	13.50 (1.88)	5.19	<0.001	1.55

Small effect size, $d = 0.2$–0.3; medium effect size, $d = 0.5$; large effect size, $d = 0.08$ (Cohen, 1988)
JK junior Kindergarten, *SK* senior Kindergarten

Table 4 Pre-post improvements on the 3D mental rotation block task (maximum score 16)

Grade	Mean age (years)	Mean pre-test (SD)	Mean post-test (SD)	*t*	*p*	Effect size
JK ($n = 8$)	4.5	6.88 (2.75)	7.25 (1.28)	0.40	0.703	0.17
SK ($n = 7$)	5.7	7.00 (1.73)	9.00 (2.00)	2.76	0.033	1.07
1 ($n = 11$)	6.8	8.64 (2.77)	9.72 (3.07)	1.42	0.186	0.37
2 ($n = 13$)	7.5	10.08 (3.15)	11.69 (3.40)	2.63	0.022	0.49

Small effect size, $d = 0.2$–0.3; medium effect size, $d = 0.5$; large effect size, $d = 0.08$ (Cohen, 1988)
JK junior Kindergarten, *SK* senior Kindergarten

research has shown that 3D mental rotation is more difficult than 2D mental rotation (see Jansen et al., 2013). The failure to obtain significant gains in the sample of Grade 1 students was the result of one child who scored well below his/her performance on the post-test compared to the pre-test, highlighting the limitations of small sample sizes. Significant gains were obtained when this child was removed from analysis, $t(9) = 2.52$, $p = 0.033$. Of particular interest, Table 4 illustrates that across the grades, children's post-test scores increased to a level approximating if not surpassing the pre-test performance means of the grade 1 year above them. For example, the SK students achieved a mean score of 9 at post-test, a higher score than the mean score of 8.64 achieved by the Grade 1 students at pre-test.

6.3 Examining the 2D and 3D mental rotation data together

In returning to raw scores of the pre-post student data for the 2D mental rotation task, we noted that 35 of the 38 participants (92 %) improved from pre to post and one participant achieved the same score at both time points. The raw score change rates were between 1 and 8 points out of a total 16 point score. The data in Fig. 7, organized by teacher estimations of student mathematics abilities, show that the "lowest ability group" improved by 3.35 points from pre to post, the mid-level ability group improved by 3.25 points, and the high ability mathematics group improved 2.88 points. This is an interesting trend that deserves further attention. Essentially, the group of students designated by teachers as having the lowest ability actually gained the most during the

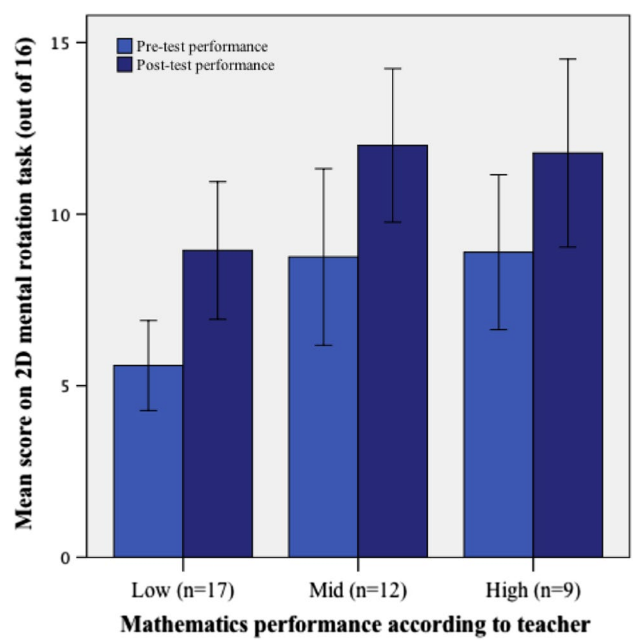


Fig. 7 Mean performance on the 2D mental rotation task according to mathematics ability level. Ability levels were determined by each child's teacher prior to the Lesson Study intervention

intervention timeframe. This leads to a host of questions including considering what the effects of a longer intervention period might have been—would the gap in spatial abilities continue to narrow? Also interesting to note is that teacher estimations of low student abilities in overall mathematics matched with their spatial scores on the 2D and 3D mental rotation tasks. That is, those students who were

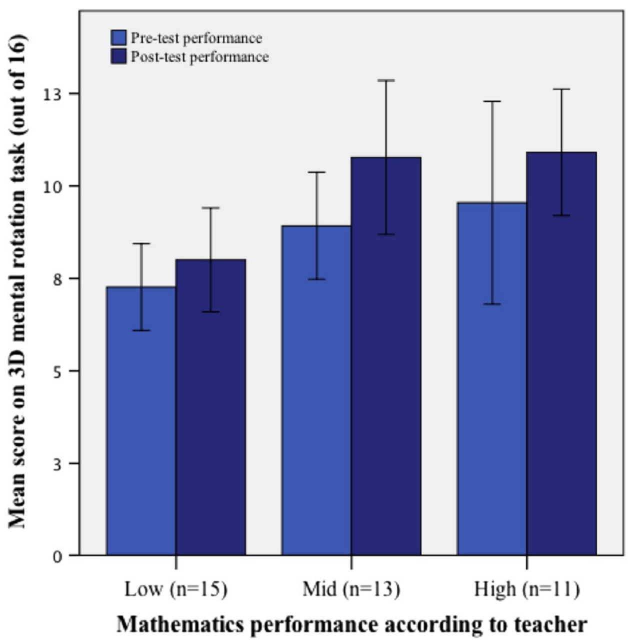


Fig. 8 Mean performance on the 3D mental rotation block task according to mathematics ability level. Ability levels were determined by each child's teacher prior to the Lesson Study intervention

identified as struggling with mathematics were the lowest scoring group on the spatial tasks. One question the data now poses is whether the student improvements in spatial ability will transfer to mathematics performance overall. We are currently in the process of tracking this same population of students to explore this conjecture further.

In returning to the raw data of the 3D mental rotation task (see Fig. 8) we see that 25 of 39 students (64 %) improved from pre to post. Four children achieved the same score at both time points. Again, there was a large range of improvement from 1 to 7 points of the raw score total out of 16 points, but these improvements were not as large nor consistent as the improvements seen on the 2D task. One explanation for this variation is that the 2D mental rotation tasks are generally considered more accessible (Bauer & Jolicoeur, 1996; Hoyek, Collet, Fargier, & Guillot, 2012; Shepard & Metzler, 1988). The trend of teacher assignment to low-, mid-, and high-achieving students held fast in that those students who were assessed as struggling the most, were also the lowest in their spatial abilities. However, the trend observed in the 2D task, in which the low ability mathematics children improved the most, did not hold up for the 3D mental rotation task—the low children improved by 0.77, the mid improved by 1.85, and the high by 1.4. Again, given that 3D mental rotations are typically considered more challenging than 2D mental rotations, we may need to account for the degree of difficulty associated with the 3D mental rotation block task and its demand for more cognitive resources.

Overall, we see the same trends across tasks emerging and that students of all ability levels demonstrated improvement.

When considering the 2D and 3D results together, these data demonstrate that accelerated development of 2D and 3D mental rotation skills is possible over a relatively short period of time (i.e., a 4-month period). This finding in itself is noteworthy, given some previous conjectures that mental rotation is a relatively stable and fixed-trait of intelligence (Johnson & Bouchard, 2005). On the contrary, our data confirm that educational interventions focused on fostering children's spatial thinking are effective at improving young children's skills at mentally rotating 2D and 3D figures.

Although it is unlikely that the observed intervention effects were due to taking the same test twice (i.e., test–retest effects), this possibility cannot be ruled out without the inclusion of a control group. With that said, our in-class observations and video analyses of children's learning through the various activities and lessons that the teacher-researcher team planned and implemented (see Table 2) provide reasons to be optimistic that the intervention was at least in part responsible for the observed student gains. Control population data are currently being collected to compare to these and other intervention study data.

6.4 Opportunity to learn and shifts in estimation of student abilities

Students of the participating teachers in School A were provided with extensive opportunities to engage with spatial reasoning tasks in different contexts such as whole group, small group, play stations and centers, and on an individual bases over 4 months. The teacher-researcher team generated a wealth of exploratory tasks for their students, tried variations of these tasks based on observed student needs, and readily shared customized materials, as well as their observations of student thinking with one another. Students were exposed to geometry and spatial reasoning tasks that far exceeded the norms in Ontario classrooms. Opportunities for students to learn were intensified in quantity (at least 14 tasks over 4 months) and in breadth (novel tasks that moved beyond shape naming and classification—posted at http://tmerc.ca/m4yc/).

At the end of the study, the School A teachers made a collective list to summarize shifts in their estimation of what mathematics their students were capable of:

- Students more capable than anticipated at 3D mental rotation
- Lower achieving mathematics students surprised us, performed same or similar to higher students
- Students naturally using language, "Flip" and "Turn" with no previous prior instruction

- Students attaching meaning to 3D figures (gestalt) e.g., it looks like a 'llama', 'chair', 'the letter W'
- Girls performed same as boys (field notes).

Of particular interest in this teacher-generated list is the change in teachers' estimations of what their young students were capable of (particularly in the area of mental rotations). The teachers recognized at the end of the study that some of the students they had previously identified as lower achieving students in their classes were as capable of improvement as those students previously identified as having high mathematics abilities once provided with opportunities to learn. During the focus group interview, the teachers attributed these shifts in their understanding to two aspects of the Lesson Study work: First, they described the immediate classroom application of the interventions—"it was immediacy in terms of the activity and the lesson going on". Second, they noted the importance of moving beyond the classroom walls to collaborate with one another formally and informally—"Just the collaboration that went on well before the meetings. We would pass the materials throughout the division and it was just, you know, to see a grade 2 teacher working with kindergarten and back and forth. The materials were flowing."

The promising student gains in mental rotation abilities were attributed, by the teacher participants, to lesson study activity, to math content learning, and to the increase in teacher expectations of students in the M4YC program. As one of the teachers at School A stated

> I think my biggest take-away, other than all of the geometry and the math, was that "no-ceiling" curriculum. And that for me, in Kindergarten, I should expose and push and provide opportunity and problems so that those students that are already strong have a chance to be pushed even further and those that maybe don't gravitate to playing and exploring their world geometrically have more opportunity than me just doing, "circle, square, triangle." I need to find or create better learning opportunities for my students so that they will develop that spatial reasoning. Because if I just teach them to name things, the vocabulary, that's not going to develop their spatial sense at all. (teacher focus group interview)

7 Summary

The goal of the Mathematics for Young Children program is to increase the level of mathematics learning and activity for both teachers and their young learners from ages 4 through 7. The focus of this work is largely on improving our (teachers and researchers) understanding of the complexities of spatial reasoning and mental rotations in particular, and how that translates to classroom learning contexts. This work has led to practical classroom resources that have been field-tested in Ontario schools, and shared widely through web spaces (http://www.tmerc.ca).

The results of this work to date are promising and suggest that providing varied opportunities for young students to engage in dynamic spatial tasks has a positive impact on their abilities to perform 2D and 3D mental rotations. The data presented in this paper focus squarely on mental rotation as a key metric of spatial skills. They support findings that mental rotation abilities are malleable, and that with practice, they can be improved. The students from School A came from an impoverished community with relatively low provincial test scores (twelve percentage points behind the provincial average); thus it is particularly encouraging that these gains were achieved. Our data also suggest that the ability to improve spatial reasoning in the form of mental rotations may begin at an earlier age than previously measured and can be achieved in the context of classroom tasks and lessons. The post-test scores observed across the grade levels indicated steady improvements in both 2D and 3D mental rotation skills. The results of this study also showed that it is possible to accelerate the growth of young children's mental rotation skills through a variety of teacher delivered lessons and activities. This is an important finding for several reasons. First, although many studies have provided evidence for the malleability of spatial thinking, these results are largely derived from carefully controlled studies with extremely precise experimental manipulations. The current study offers a different approach to teaching spatial thinking and demonstrates that spatial thinking is malleable in 'noisy' but authentic classroom contexts. So, while our study design does not allow us to pinpoint the mechanism or specific activities that led to improvements in children's mental rotation skills, we did show that early years teachers are effective at both designing and implementing a highly effective spatial curriculum. In this way, our study is an ecologically valid approach to spatial learning and bridges a previous gap between the work of mathematics education researchers and cognitive psychology researchers. Second, this study is of importance for its potential support to overall mathematics learning. Given that spatial thinking is intimately linked to success in overall mathematics, we hypothesize that improving children's spatial thinking can have a "two-for-one" effect where improvements in spatial reasoning may also be seen in overall mathematics (see also Verdine, Golinkoff, Hirsh-Pasek, & Newcombe, 2014). Future research efforts are needed to determine the extent to which improving spatial learning generalizes to gains in mathematics performance.

There were two main limitations of this study. The first was the absence of a control group. It is possible that the gains observed over the 4-month Lesson Study period were a result of taking the same test twice (i.e., test–retest effects). Another possibility is that the associated gains in mental rotation were a result of natural development. These limitations notwithstanding, it seems unlikely that the large gains reported here were not at least in part due to the intervention. The finding that post-test scores were comparable to the pre-test scores of students one grade level ahead speaks to the strength of the intervention. Such large gains achieved over a relatively short period of time seem unlikely products of test–retest or natural development. Nonetheless, as mentioned above, the current research study will be strengthened with the inclusion of a control group. The second limitation was sample size. Given the classroom-embedded and complex nature of Lesson Study and this spatial reasoning intervention, it is not surprising that the *n* is low and is responsible for some particular inconsistencies in the data at present. Data analyses are continuing in this longitudinal study with increasingly larger sample sizes and comparable control groups to examine developmental readiness of young children for 3D and 2D mental rotation tasks, gender and SES disaggregated differences, as well as levels of increased intensity of student self-corrections pre to post as evidence of cognitive engagement. The project has also been extended to track students over 3 years in an effort to generate a reliable database of mathematics and spatial reasoning development over time.

Our focus on spatial reasoning has expanded our theoretical and practical conceptions of what might be included in the geometry curriculum for young children. For example, the geometry and spatial reasoning tasks implemented in School A were particularly dynamic in nature (objects transforming and rotating in space, both in the mind's eye and in our physical 3D world) and moved well beyond what the teachers of the study had previously considered as geometry (both theoretically and in their classroom programs). The teachers were enthused to explore geometry in new ways that increased their understanding of 'what is possible'. Because of the classroom-embedded nature of Lesson Study, the teacher-researcher team was able to observe students, build on these young children's strengths, design and test out a range of tasks to meet their needs, and to examine the benefits of these tasks. Thus the Math for Young Children program is achieving the two purposes outlined by Stylianides and Stylianides (2013) of both improving classroom practice and deepening our theoretical understandings of classroom phenomena related to these problems of practice.

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