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Understanding gaps in research networks: using "spatial reasoning" as a window into the importance of networked educational research

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Abstract This paper finds its origins in a multidisciplinary research group's efforts to assemble a review of research in order to better appreciate how "spatial reasoning" is understood and investigated across academic disciplines. We first collaborated to create a historical map of the development of spatial reasoning across key disciplines over the last century. The map informed the structure of our citation search and oriented an examination of connection across disciplines. Next, we undertook a network analysis that was based on highly cited articles in a broad range of domains. Several *connection gaps*—that is, apparent blockages, one-way flows, and other limitations on communications among disciplines—were identified in our network analysis, and it was apparent that these connection gaps may be frustrating efforts to understand the conceptual complexity and the educational significance of

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spatial reasoning. While these gaps occur between the academic disciplines that we evaluated, we selected a few examples for closer analysis. To illustrate how this lack of flow can limit development of the field of mathematics education, we selected cases where it is evident that researchers in mathematics education are not incorporating the important work of mathematicians, psychologists, and neuroscientists—and vice versa. Ultimately, we argue, a more pronounced emphasis on transdisciplinary (versus multidisciplinary or interdisciplinary) research might be timely, and perhaps even necessary, in the evolution of educational research.

Keywords Spatial reasoning · Network analysis · Mathematics education · Transdisciplinary approach

1 Introduction

Spatial reasoning and its contribution to mathematical cognition has been a consistent topic of interest among mathematics educators for some decades now (e.g., Bishop, 1980; Gattegno, 1965; Presmeg, 1986; Tahta, 1990). Over the past several years, however, attention toward the topic has been increasing rapidly alongside realizations that spatial reasoning abilities are vitally entangled with a great many other concerns—including, for example, curriculum reconceptualization, classroom organization, teaching strategies, and career demands in an information economy.

Attentive to this growth in interest, our Spatial Reasoning Study Group (SRSG) first gathered in 2012 to explore possible research synergies. Since then, the SRSG has been engaged in multidisciplinary (and multi-national) research, drawing together expertise from Mathematics Education, [1] Psychology, Mathematics, Cognitive Science, and Philosophy, with the main aim of studying the role of spatial reasoning in mathematics teaching and learning (Bruce, Moss, Sinclair, Whiteley, Okamoto, McGarvey, & Davis, 2013; Sinclair & Bruce, 2014; Davis & Spatial Reasoning Study Group, 2015).

Importantly, the multidisciplinary makeup of our group was a deliberate decision. The move was in response to a growing acknowledgement that the pursuit of knowledge in academic communities might be amplified by bringing together diverse sorts of disciplinary expertise. This is especially true when the goal of research is to "resolve real world or complex problems, to provide different perspectives on problems, [and] to create comprehensive research questions" (Choi & Pak, 2006, p. 351)—that is, for example, to investigate the sorts of situated, adaptive phenomena that occupy the interests of many Mathematics Education researchers. Such research can be challenging to undertake, particularly where the collaboration requires adapting to specialized disciplinary discourses (see Spanner, 2001).

Of course, this point is not new within Education—which, arguably, is a necessarily multidisciplinary domain. Not only was Education originally rooted in a convergence of Psychology, Sociology, and Anthropology, in a strong sense Education links to all disciplines in their needs to perpetuate themselves. That said, the simultaneous presence of multiple disciplines does not constitute a transdisciplinary approach—and this realization has prompted

[1] Because this writing is concerned with research within and communications among a number of disciplines, we have adopted the convention of capitalizing the names of those disciplines whenever we refer to recognized domains of inquiry. This convention is useful to distinguish between, for example, the field of Mathematics and the activity of learning mathematics.

some to propose distinctions among "multidisciplinary," "interdisciplinary," and "transdisciplinary" research attitudes. In coarse terms, the differences among these orientations might be characterized by their principal emphases: respectively, combining monologues, creating dialogues, and sustaining holistic conversations (Choi & Pak, 2006). One goal of this paper is to show that transdisciplinary approaches—that is, holistic, problem-rooted inquiries that seek to integrate diverse expertise from across domains—are necessary if progress is to be made on important, complex problems in the teaching and learning of mathematics. Transdisciplinary inquiry aims to transcend the insights and solutions available from singular disciplinary positionings, "while at the same time maintaining the advantages of creativity and initiative peculiar to each specific field of knowledge" (Lattanzi, 1998, p. 13).

That goal springs from an early realization among members of the SRSG. As we grappled with the demands of communicating our respective understandings of spatial reasoning to one another, it was evident that the topic is, simultaneously, a focus of extensive research *within* a great many disciplines, yet not a topic of significant discussion *across* most of those disciplines. Embracing the notion that Mathematics Education is by definition a multidisciplinary domain, we thus saw ourselves as well positioned to take on the paired tasks of, first, assembling a review of research in order to better appreciate how spatial reasoning is understood and investigated across academic disciplines and, second, of analyzing the structures of interactivity among disciplines around the topic of spatial reasoning with a view toward offering commentary on the broader matter of transdisciplinary collaborations.

This writing is, in effect, an account of one element in that project. It begins with an introduction to the topic of spatial reasoning, which is followed by a brief report on our network analysis of highly cited articles in a broad (although not comprehensive) range of disciplines. Through that analysis, we endeavor to foreground how lack of information flow can limit not just the development of an idea, but the development of a field—points that we illustrate by looking more closely at a few specific "connection gaps"—that is, apparent blockages, one-way flows, and other limitations on communications among disciplines. We use those instances to highlight some of the mutual limitations that can arise in multiple disciplines' failures to communicate and collaborate. At the same time, we argue that researchers are not without agency. Strategies can be developed to enhance cross-disciplinary communications and, in the process, expand possibilities for truly transdisciplinary inquiry.

2 Background on spatial reasoning

As noted, spatial reasoning has been a topic of interest within Mathematics Education since the 1970s. Recently, it has risen to considerable prominence in the community, in large part because it has been recognized as a vital component of studies of and careers involving Science, Technology, Engineering, and Mathematics (STEM) (Newcombe & Shipley, 2015; Wai, Lubinski, & Benbow, 2009). Further impetus has come from the realization that spatial reasoning is learnable. There is mounting evidence that spatial reasoning and related competencies are malleable at any age and for both genders, and that the associated skills are strongly correlated to achievement across not just STEM domains but all subject areas (Newcombe, 2010, 2013).

Even more fundamentally, however, interest in spatial reasoning seems to have paralleled a growing embrace of the assertion that humans are embodied, situated beings (Lakoff & Núñez, 2000). Humanity has evolved and humans develop in the context of a four-dimensional world: As one proceeds in time through childhood and adolescence into adulthood, one is continuously learning and problem solving in the spatial environs of height, length, and depth. A remarkable physiology enables this journey, especially evident in the interplay of a predominant verticality (in stance) and a predominant horizontality (in navigated spaces)—which, in turn, is manifest in cognitive abilities and conceptual development. When an individual explicitly engages in spatial reasoning, she or he is working with a spatial model of some phenomenon of interest. This interaction may involve the spatial properties of an object or spatial relations between objects, and change over time in spatial coordinates may be relevant (Uttal et al., 2013). In the wildlands of the classroom, children rapidly switch between modes of spatial analysis as they problem solve, with impressive results including surmounting spatially complex Lego™ robotics challenges with little direct instruction from teachers (Khan, Francis, & Davis, 2015).

Examples of spatial reasoning thus include locating, orienting, decomposing/recomposing, balancing, patterning diagramming, navigating, comparing, scaling, transforming, and seeing symmetry. From these, we derived our preliminary definition of spatial reasoning as the ability to recognize and (mentally) manipulate the spatial properties of objects and the spatial relations among objects.[2]

This definition, of course, is profoundly influenced by the SRSG's specific interest in learning and teaching mathematics, and it is thus highly reflective of other descriptions encountered in the field of Mathematics Education. As our discussions moved further afield, however, there were immediate indications that other domains have somewhat different foci, evident in part by the names they use—e.g., *spatial ability*, *spatial sense*, *spatial intelligence*, or *spatiality*, to name a few. When delving into relevant literatures across some of these domains, it became evident that the above definition was insufficient to embrace the diversity of foci that have arisen in the long, tangled history of the concept. This finding is in line with the observations of other researchers such as Hegarty and Waller (2005), and the point was driven home for the SRSG when we attempted a collaborative expert-based, historical map of spatial reasoning (see Fig. 1). The exercise involved working in subgroups of two and three to identify key thinkers, researchers, and key paradigms in the history of the study of spatial reasoning. The small group structure allowed for checks and balances in determining the importance of a given paradigm but also ensured overlap between groups to confirm that these perspectives were significant based on collective knowledge of the fields.

Our process in generating the historical map pointed to some important realizations. First, it confirmed the viability of spatial reasoning being taken up as a transdisciplinary area of research. We identified seminal research in a diverse range of disciplines across the physical and social sciences. Second, as we reviewed the literature, we attended to the bibliographic references and noted that disciplines were primarily self-citing. There was limited crossover in citations between the sciences and arts, and even between seemingly related domains such as Mathematics Education and Mathematics. And third, our search revealed the broad range of terms related to the construct of spatial reasoning within the different fields of study. We considered that the diversity of keywords might limit researchers' abilities to locate related studies in different disciplines. This third finding was troubling as we recognized in our map

[2] As the focus of this article is transdisciplinarity, illustrated through the instance of spatial reasoning, it is beyond our current purposes to critique or elaborate this preliminary definition. However, we have done so elsewhere. See, in particular, the closing chapter of Davis, Francis, and Drefs (2015).

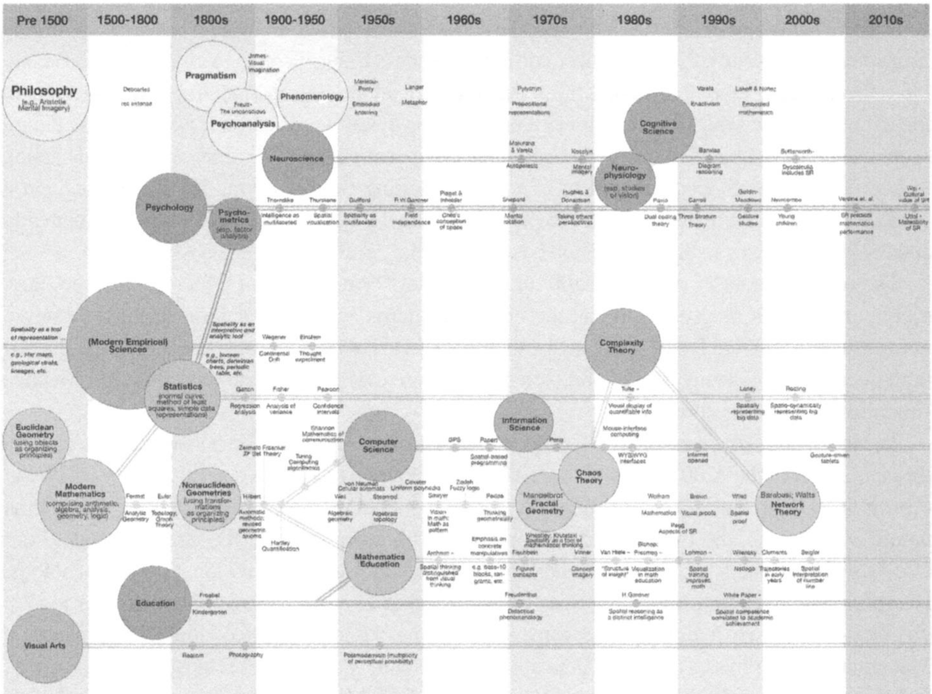


Fig. 1 An attempt to trace the historical emergence of the construct of spatial reasoning

the predominance of literature in Mathematics, Mathematics Education, and Psychology—our group's areas of expertise. The map may not be particularly reflective of the broader landscape where we noted, for example, Statistics and Visual Arts are under-represented.

Despite our efforts at being thorough and objective, there are multiple limitations with depicting the rich history of spatial reasoning as we did through our original mapping. The physical placement of each domain on the figure is somewhat arbitrary and the connections among domains are not readily apparent on this linearized representation. The discrete timelines do not reflect the complexity of cross-domain communication and influence. In order to investigate our findings in more depth and ameliorate some of the limitations, we turned to network analysis.

3 Network analysis as a methodology for examining relations

With the ultimate goal of incorporating and extending the research from multiple disciplines into our own research in Mathematics Education, we turned to network analysis to understand current communication patterns across fields (e.g., Borgatti, Mehra, Brass, & Labianca, 2009). In particular, we drew upon citation analysis as a way to represent the social network of academic authors who are connected through citations (Fu, Song, & Chiu, 2014). Citation analysis offers powerful and well-tested tools for quantitatively measuring and qualitatively mapping the citation patterns between and among scholarly works. We sought to visually represent the current state of interdisciplinarity related to spatial reasoning research and identify possible opportunities for transdisciplinary inquiry (i.e., deliberate, problem-focused collaborative efforts among experts from different disciplines).

To limit the scope of analysis here, we selected disciplines most relevant to our work including Education, Psychology, Neuroscience, and Mathematics. Using Scopus, a citation database, we identified refereed journal articles from 2000 to present addressing spatially relevant topics. In an initial and iterative search, we identified a broad range of keywords across disciplines, including spatial perception, spatial skills, spatial structure, perspective taking, symmetry, and visualization (see Bruce, Davis, Sinclair, & and the Spatial Reasoning Study Group, 2015). To maintain a balance across disciplines, we then selected approximately 1500–2000 of the most frequently cited refereed articles in each discipline based on the keywords.

The resulting dataset included 7200 unique articles.[3] We employed bibliographic coupling, a citation analysis measure, to examine citation patterns across the dataset using the journal name as the unit of analysis (Zhao & Strotmann, 2015). Two works were considered coupled if both cited the same source. The more sources the two texts have in common in their reference lists, the stronger the coupling.

The results from the analysis are displayed in the distance-based citation network in Fig. 2. First, the citations from the 7200 articles produced 6 colored journal clusters as computed by VOSviewer (Van Eck & Waltman, 2016). In brief, the greater the relation in journals, the closer the journal names appear; also, the more coupled a journal is with other journals, the larger its node. Four closely related clusters (red, light blue, dark blue, and yellow) are primarily journals in Psychology, Neuroscience, and Neurology. This close grouping indicates a high degree of coupling or journal co-citation. The two remaining and somewhat distant clusters consist of journals primarily in Mathematics and Physics (red) and Education (green) with Mathematics Education journals (e.g., ZDM, Educational Studies in Mathematics, Journal of Mathematical Behavior) being most prominent in this latter grouping. The noticeable disconnect between the green-Education cluster and the other five clusters indicates the lack of journal coupling or bidirectional citation.

The network provides a macro analysis illustrating that the bidirectional information flow of spatially relevant research between Education and the disciplines of Psychology, Neuroscience, and Mathematics is weak. In order to engage in a more focused or micro-level investigation, we examined the high-frequency keywords and highly cited research within and across disciplines in the network analysis. We listed areas of research that were relevant to Mathematics Education; then, the list was narrowed further based on the SRSG members' areas of familiarity and expertise; finally, we selected a representative case related to each of the disciplines within our network: (1) perspective taking in Psychology and Mathematics Education; (2) symmetry in Mathematics and Mathematics Education; and (3) pattern recognition in Neuroscience and Mathematics Education.

4 Research areas in mathematics learning as possible occasions for transdisciplinary inquiry

In this section, we examine whether multidisciplinary or interdisciplinary research exists and the possibility for, and obstacles to, transdisciplinary inquiry within the specific area of study. These cases are drawn from Psychology, Mathematics, and Neuroscience, respectively.

[3] All journals within the Scopus database are classified in one or more major and minor subject areas. Neuroscience, Mathematics, and Psychology are each considered major subject areas. Education is a minor subject area within Social Sciences. Mathematics Education journals are typically classified under the major subjects of Mathematics and Social Sciences and under the minor subjects of Education and Applied Mathematics, respectively.

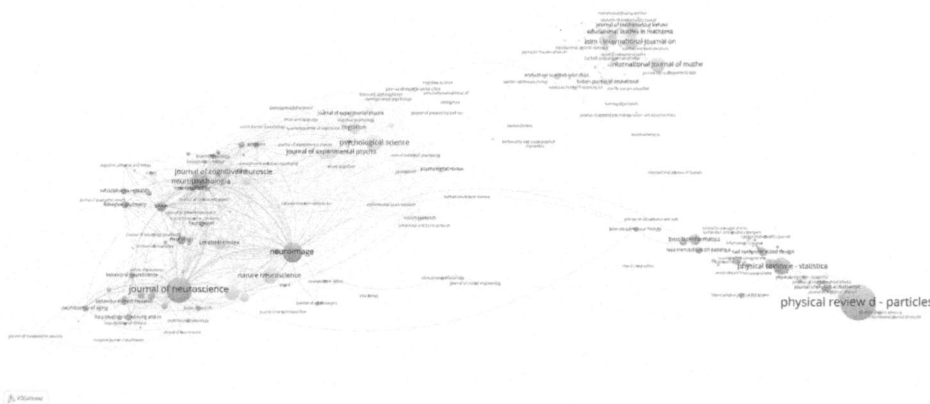


Fig. 2 A mapping of spatially relevant research in Education, Psychology, Neuroscience, and Mathematics designated journals (date of analysis: 2016.05.22)

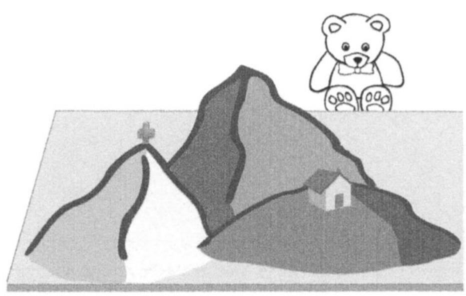
4.1 Case one: perspective taking—a Psychology–Mathematics Education connection gap

In Psychology, "perspective taking" is a cognitive construct that originated in the seminal work of Jean Piaget (Piaget & Inhelder, 1948/1967). In Piaget's classical test, "The Three Mountains Task," the child is presented with a landscape scene and asked to describe it from other perspectives (Fig. 3), requiring the child to temporarily abandon their own viewpoint and instead imagine the view from a different physical location.

The construct is often associated with a child's moral development; that is, the developmental shift from egocentrism to considering another's viewpoint (Piaget, 1932/1997). More recently, variations of the task have contributed to richer understandings of children's spatial reasoning. For example, Frick, Möhring, and Newcombe (2014) designed a task involving a three-dimensional setup, as shown in Fig. 4. The children were asked which character (if any) had taken the picture shown above the eye icon. Perspective taking, within the discipline of psychology, is a cognitive skill that pervades everyday behavior; humans use perspective taking to make sense of the world physically and spatially (Frick, Möhring, & Newcombe, 2014).

In contrast to Psychology, Mathematics Education research tends to ignore perspective taking; yet, when viewed through a lens of mathematical learning, perspective taking tasks, such as Fig. 4, are noticeably relevant to spatial thinking. In an examination of curriculum standards, we begin to recognize that a form of perspective taking is prevalent in many

Fig. 3 Piaget's "Three Mountains Task," in which the viewer is asked to describe the scene from the perspective of the bear



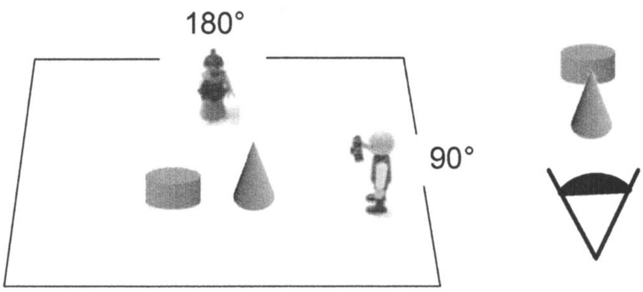


Fig. 4 Perspective taking task used in Psychology

outcomes such as sketching three-dimensional structures from multiple perspectives as illustrated by a Grade 6 expectation in the Ontario curriculum:

> build three-dimensional models using connecting cubes, given isometric sketches or different views (i.e., top, side, front) of the structure. (Sample problem: Given the top, side, and front views of a structure, build it using the smallest number of cubes possible.) (Ontario Ministry of Education, 2005, p. 92)

While this treatment of perspective taking is somewhat limited and reduces the complexities of the construct to isolated skills, it is a useful example to draw attention to the pervasiveness of perspective taking in many aspects of mathematics through activities such as drawing, composing figures, decomposing shapes, navigating, and mapping.

The construct of perspective taking within a Mathematics Education context was the subject of a recent article, by Van den Heuvel-Panhuizen, Elia, and Robitzch (2015). Their study examined the performance of kindergarten children on imaginary perspective taking (IPT). They found that IPT performance was significantly related to mathematics ability. As part of the study, the authors devised a set of tasks, adapted to school mathematics and assessed their participants' IPT. The tasks shown in Fig. 5 reflect Mathematics Education content in that they involve shape perception and manipulation, topics of relevance to the elementary school curriculum.

The discussion above begins to illustrate how highly complementary literatures around perspective taking are in Mathematics Education and in Psychology. While we recognize the commonalities, we also acknowledge the potential discrepancies in communication between the discourses of Mathematics Education and Psychology. The experiment-based work in Psychology places attention on issues such as task reliability and standardized administration. The two items in Fig. 5 may lack

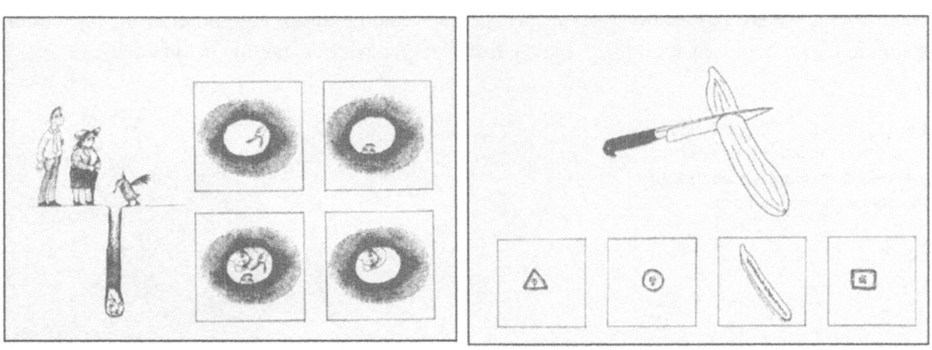


Fig. 5 Two items from Van den Heuvel-Panhuizen et al., devised by Mathematics Education researchers to measure IPT (used with permission)

the particular type of rigor used for validation within the discipline of Psychology; yet, education researchers must attend to issues of contextual validity and adaptive response, and they may find the psychological perspective taking tasks to be too narrowly focused. Thus, Psychology may not value the findings of Mathematics Education researchers, while the Mathematics Education researchers may have some difficulty in finding relevance in the way in which constructs are defined and assessed in Psychology studies.

If a problem-driven transdisciplinary attitude can be taken to address the gap between disciplines, these understandings on perspective taking might converge to address some of the pragmatic needs within Mathematics Education to help learners develop strategies and skills that support achievement.

4.2 Case two: symmetry—a Mathematics–Mathematics Education connection gap

Our second case explores the topic of symmetry as it relates to Mathematics and Mathematics Education. Within Mathematics, symmetry is a broad, dynamic construct, and was one of the most frequent keywords in our network analysis. Many mathematicians "do symmetry" with algebra and transformations—that is, through symbol-based formulas rather than through images. In fact, the uses and interpretations of symmetry in Mathematics are so extensive that it makes little sense to attempt a survey. Branches of Mathematics in which symmetry plays prominent roles include calculus, linear algebra, differential equations, group theory, set theory, topology, graph theory, probability, combinatorics, and elementary arithmetic including balancing equations. Even within geometry, mirror symmetry and rotational symmetries do not begin to cover the many types used to analyze images and figures, such as translational symmetry, glide reflection symmetry, roto-reflection symmetry, helical symmetry, double rotation symmetry, periodic symmetry, non-isometric symmetries, projective symmetry, and scale symmetry. In fact, one modern definition of "a geometry" involves the symmetries of a collection of objects—that is, the invariances of a particular set of forms under a specified group of transformations. That definition is reflected in the broader mathematical definition of symmetry, as a type of invariance. Symmetry is a property that does not change under a group or groupoid of transformations, where a transformation is understood as a mapping of an object onto itself that preserves the structure. In general, every kind of mathematical structure will have its own kind of symmetries. In an attempt to synthesize the many interpretations and applications of symmetry in (Pure) Mathematics, Yanofsky and Zelcer (2015, p. 10) offered a visual definition of symmetry as shown in Fig. 6. The diagram intends to show how a symmetry requires an invertible transformation (hence arrows in both directions, from A to B and from B to A) that takes a true statement to a true statement under evaluation at elements of the sample space. (Since the transformation is reversible, it also maps false statements onto false statements.)

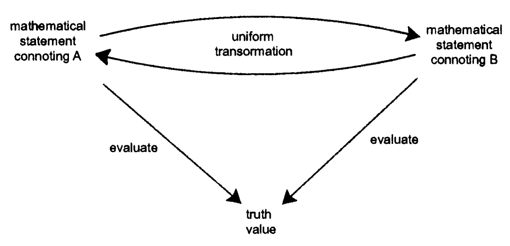


Fig. 6 A visual "definition" of symmetry within Mathematics (from Yanofsky & Zelcer, 2015)

By way of a more specific example, Pierre Curie's principle, which asserts that if the input to the event is symmetric, then the output will have the same symmetry, can be illustrated through Fig. 7. In the example of solving equations, the conclusion is that the solution set is symmetric (not that each of the solutions is symmetric).

Within mathematics teaching and learning, the word *symmetry* summons images of butterflies and geometric designs—that is, figures that can be sliced into a pair of mirror images. For some, it also calls up a set of images that can be "rotated onto themselves," such as the yin–yang symbol and certain spirals. No doubt these understandings are rooted in the fact that interpretations of symmetry within the field Mathematics Education rarely go beyond the applications of folding and rotating.

As one curricular example, in the Kindergarten to Grade 9 Western and Northern Canadian Protocols (WNCP, 2007[4]), the topic of symmetry is only mentioned once in Grade 4 and once in Grade 9. Generally, classroom math programs focus on mirror symmetry, and to a lesser extent on rotational symmetry. Across almost all instances of school mathematics, symmetry is treated as a static property of two-dimensional images—that is, a quality that is already manifest in a stable form rather than, for example, the result of multiple actions or a dynamic property of emerging phenomena. In essence, the common treatment of symmetry in school mathematics presses students to attend to non-moving objects or parts of objects.

Within Mathematics Education, a small body of research on symmetry has emerged to involve the use of computer-based tools, which enable the kind of dynamic transformations of shape that are not typically available in everyday experiences. Examples include early research involving Logo (Clements, Battista, & Sarama, 2001), as well as a more recent study involving Sketchpad (Ng & Sinclair, 2015). The latter explicitly draws on insights from Mathematics, both to build rationale for the significance of symmetry in school mathematics and to underscore the importance of linking symmetry to transformations.

Besides the few examples described above, Mathematics Education has not adequately drawn on Mathematics in the area of symmetry and more broadly spatial reasoning. Extending our exploration to other disciplines, we note that symmetry is central in the Arts. A biologically rooted interpretation of symmetry has also been argued by Weyl (cited in McManus, 2005) who asserted that, from an artistic perspective, *symmetric* means something like "well-proportioned, well-balanced," and *symmetry* denotes a concordance of several parts by which they integrate into a whole. Beyond artists, many understand that beauty is bound up with symmetry. Such assertions, in fact, are at the core of the transdisciplinary domain of Neuroaesthetics (Schott, 2015). Similarly, we note that research in Psychology shows that children come to school with an already strong capacity for identifying symmetry (Bryant, 2008), suggesting the potential for much more in-depth learning in this area.

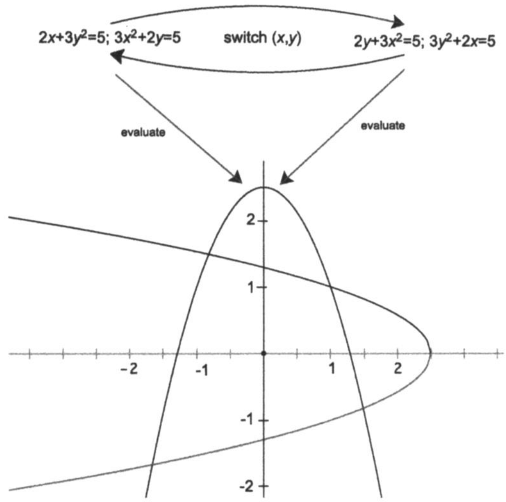
A transdisciplinary Mathematics and Mathematics Education approach to studying symmetry is not important so that it can become a curriculum topic add-on; instead, symmetry in the service of spatializing the mathematics curriculum might allow us to conceive of curriculum in terms of preparing children to live in their world.

4.3 Case three: Educational Neuroscience—addressing a Neuroscience–Mathematics Education connection gap

In our network mapping process, we identified a significant number of Neuroscience research papers relating to spatial reasoning. In particular, a major field of research in Neuroscience

[4] Available through: http://www.wncp.ca/.

Fig. 7 A visual interpretation of Curie's principle



involves studies of pattern recognition and perception in students with special needs, such as those who are high functioning on the autism spectrum. For example, neuroscientists Mottron, Dawson, and Soulières (2009) proposed that the "enhanced detection of patterns, including similarity within and among patterns, is one of the mechanisms responsible for operations on human codes" (p. 1385). Recent studies in Neuroscience also suggest that children with autism have exceptional ability to process local and global information simultaneously, an ability in normal development that must be conducted serially (Trevarthen & Delafield-Butt, 2013). Children with autism have recently been shown to demonstrate an exceptional ability to inhibit background noise in a perceptual field in order to perceive meaningful spatial patterns simultaneously, including vertical, oblique, and horizontal symmetry (Perreault, Gurnsey, Dawson, Mottron, & Bertone, 2011). Although this research comes from Neuroscience, rather than Education, it points to a critical opportunity for Mathematics Education to expand its horizon by incorporating knowledge from another domain: taking this new knowledge from Neuroscience to Education has immediate and relevant implications for both Psychology and Mathematics Educators.

In Mathematics Education research, the role of visual pattern recognition and perception has emerged as a critical focus of research in early learning (e.g., McGarvey, 2012; Papic, Mulligan, & Mitchelmore, 2011; Warren & Cooper, 2008). For example, Mulligan and Mitchelmore (2009) have argued that pattern recognition and perception are inextricably linked to the structural development of mathematical concepts. They proposed a new construct, Awareness of Mathematical Pattern and Structure (AMPS), which their research has shown to generalize across early mathematical concepts, be reliably measured, and correlate with mathematical understanding.

Interestingly, both sets of studies focus on pattern recognition and forming abstractions and generalizations from patterns, but neither has cited the other. There appears, in fact, to be overlapping terminology. For example, "highest level of internal structure" (Mottron et al., 2009) seems to be comparable with "highest level of structural development" (Mulligan & Mitchelmore, 2009). Another parallel in relation to this construct is that both sets of studies also focused on the notion of structures based on units. For example, Mulligan and colleagues (e.g., Mulligan & Mitchelmore, 2013) referred to units in sequences involving repetitions, growing patterns and numbers, and equal spacing, such as partitioning of lengths and constructing units of measure. In Mottron et al. (2009), the development of stored information is also referred to as built from perceived units or structures, via reintegration, completion, or filling-in of missing information.

We did, however, identify a recent study by Tsang, Blair, Bofferding, and Schwartz (2015) which provides a compelling example of a more transdisciplinary approach. Drawing heavily from research in Neuroscience, Psychology, and Mathematics Education, the authors sought to answer how basic findings from Neuroscience and research on the psychology of perception can be used to guide the design of mathematics classroom instruction. More specifically, the authors examined whether our basic human propensity to appreciate and recognize symmetry might be leveraged in such a way as to help young children learn to "see" the inherent symmetry of the set of integers. For example, through a series of behavioral experiments, Varma and Schwartz (2011) found that children do not readily represent negative integers as reflections of positive integers. Instead, children tend to rely on rules that largely ignore the underlying symmetrical structure of integers (e.g., "positive numbers are greater than negative numbers"). In parallel, a series of neuroimaging studies were carried out that offered further evidence that a mature understanding of integers relies on brain regions associated with the symmetry processing of visual images (Blair, Rosenberg-Lee, Tsang, Schwartz, & Menon, 2012; Sasaki, Vanduffel, Knutsen, Tyler, & Tootell, 2005). Together, these findings from Psychology and Neuroscience provided the theoretical grounds for the design and implementation of a novel and experimental approach to teaching negative numbers to fourth-grade children (Tsang et al., 2015). The approach heavily emphasized the visual-spatial nature of integer symmetry about zero. For example, fourth-grade students were given opportunities to see the symmetrical relationship between positive and negative integers through a series of paper-folding exercises. Compared to two control groups, children in the experimental group demonstrated greater learning gains across a variety of measures, providing evidence that children incorporated symmetry into their mental representations of integers (Tsang et al., 2015).

In this example, we see evidence of an approach to research that blurs the conventional boundaries between Psychology, Neuroscience, and Mathematics Education. Impressively, researchers were not only able to integrate and utilize research findings from all three disciplines but were also able to immerse themselves in the practice of all three disciplines. This type of work speaks to the emerging transdiscipline of Mind, Brain, and Education (also called Educational Neuroscience) and provides reasons to be optimistic about future efforts of this sort (see Ansari & Coch, 2006).

Beyond spatial reasoning, there have also been major research findings from Neuroscience that have gained popularity, such as research on mirror neurons. However, to date, with a few exceptional areas in Mathematics, this Neuroscience research has not had much impact on Mathematics Education research. Christodoulou and Gaab (2008) point to some difficulties involved in connecting Education and Neuroscience, among them being that Neuroscience tends to be more *descriptive* while Education tends to be more *prescriptive*. Furthermore, neuroscientists tend to use highly operationalizable definitions of constructs such as memory, visualization, and intuition, which are often more narrow and homogeneous than the way mathematics educators or even psychologists use these terms. Similar to the other cases we have examined, the different research intentions may make co-citation across fields challenging. Neuroscience appears to be concerned with generalized principles related to the real world based on an already-operational system (Sylwester, 1995), rather than adaptations of this system to specific educational requirements. For example, Baars (1995) described the success of mathematical intuition as more likely reflecting the nervous system's excellent heuristics for discovering patterns in the world.

Despite the possible obstacles, topics in spatial reasoning are of importance to both Neuroscience and Mathematics Education. Opportunities for cross-disciplinary research are possible and have potential to inform both fields.

5 Promising new developments

Our network analysis and the specific cases we studied reveal that there are gaps in the flow of information and influence among the disciplines that have potentially complementary interests in spatial reasoning. The reasons for the connection gaps include differences in sources of research validity and outcome expectations across disciplines, unfamiliarity with bodies of research in similar or complementary constructs across disciplines, and limited awareness of research activity across relatively "distant" research domains.

In this section, we point to two initiatives that are more intentionally transdisciplinary—that is, projects in which researchers and educators have deliberately engaged the participation of experts from other disciplines in shared efforts to address complex problems. We use these instances as a platform to frame some major issues and promising possibilities for transdisciplinary inquiry involving Mathematics Education researchers.

5.1 Instance one: early number development as an opportunity for a Neuroscience–Mathematics Education conversation

With regard to the curriculum and research into early number development, the dominant paradigm in Mathematics Education has been focused on the importance of cardinality in children's development. The prevailing paradigm in Neuroscience is similar, as seen in the influential studies of Butterworth (1999) and Dehaene (2011), where almost all the tasks used in experimental situations involve attention to cardinal aspects of number (e.g., assessing which of two numerals or which of two collections of dots is greater).

Recently, Mathematics Education researchers (Coles, 2014; Sinclair & Coles, 2015) have drawn on the work of Neuroscience researchers (Lyons & Beilock, 2011; Lyons, Price, Vaessen, Blomert, & Ansari, 2014), who have challenged this dominant paradigm in Mathematics Education. Lyons and his colleagues have highlighted the distinct ways in which ordinal and cardinal aspects of number are processed in the brain in demonstrating, for example, that children's competencies with ordinality predict their abilities to do mental arithmetic. They have also shown that beginning in second grade, the ability to assess the relative order of number symbols is an increasingly strong predictor of mathematical achievement.

One preliminary educational innovation from this research has been to identify approaches to early number learning that emphasize ordinal aspects of number. For example, Coles (2014) described the use of Gattegno charts as an effective way for children to learn number by linking symbols to symbols—rather than linking symbols to objects, which is the traditional approach. Coles has also begun to feed insights gained from classroom interventions back to Lyons, who would like to formalize the intervention so that he can study whether it results in improvements on his ordinal tasks (personal communication). Similarly, Sinclair and Jackiw

(2014) have developed the multitouch app, *TouchCounts*, as a way of enabling children to work directly on symbols using tasks that emphasize ordinality. Sinclair and Coles (2015) are studying the potential of more ordinal approaches to teaching concepts that are usually cardinally focused, such as place value.

5.2 Instance two: math for young children as a deliberate Psychology–Mathematics Education conversation

Several members of the SRSG are involved in a project, known as Math for Young Children (M4YC), that exemplifies how connection gaps can be filled in ways that promote productive advances in Mathematics Education. Begun in 2011, M4YC is a collaborative project that is aimed at supporting the teaching and learning of early-years geometry and spatial reasoning. Using an adaptation of Japanese Lesson Study, the project brings together researchers, teachers, school administrators, mathematics consultants and numeracy facilitators, and Ministry of Education (Ontario, Canada) personnel.

This project began with the team of Mathematics Education researchers and teachers reviewing work in Psychology that was relevant to spatial reasoning. The team identified particular tasks that were being used in Psychology that would be appropriate for the classroom. Data from the children's activities were then shared back with prominent researchers in Psychology, such as Nora Newcombe and Susan Levine. This two-way flow of information, which included co-presentations and cross-citation, has led to the design of lessons, activities, and new resources (e.g., assessments) that support the teaching and learning of specific aspects of geometry and spatial reasoning. The project has yielded promising findings, including evidence of teacher change and growth in students' spatial and geometric reasoning (see Moss, Hawes, Naqvi, & Caswell, 2015). It has also resulted in research insights for both Mathematics Education and Psychology.

5.3 Discussion

We believe that these instances illustrate the point that, as a discipline, Mathematics Education is ideally situated to critically involve itself in the transmission and promotion of transdisciplinary research and practice. For the purposes of illustration, the relatively untapped research area of spatial reasoning provides an instructive example for how that might happen.

Of primary concern, however, is that even when efforts to create cross-disciplinary dialogue exists, unless there is purposeful effort to consider the potential obstacles, researchers may still operate within the silo of their own discipline. A clear example of this can be seen in a special issue on "Cognitive Neuroscience and Mathematics Learning" (2016) in *ZDM—The International Journal for Mathematics Education*. Ironically, although the explicit intent of this volume was to bring together research from Cognitive Neuroscience and Mathematics Education—with the ultimate goal of improving mathematics teaching and learning—the articles pay little attention to the work of mathematics educators and Mathematics Education as a whole. In fact, only a small proportion of the journal articles cited within the volume (7%, excluding books and ZDM articles) were published in Mathematics Education journals. This was even the case for topics, such as fractions, that have been the subject of rigorous and knowledge-yielding study in Mathematics Education for decades.

We would thus argue that transdisciplinary inquiry should involve (and would be supported by) deliberate efforts to close connection gaps through establishing multiple-route exchanges of information. The topic of spatial reasoning serves to illustrate this point. Psychology is replete with examples of correlational studies that seek to reveal the importance of certain psychological constructs, including spatial reasoning. For example, there is over a century of research showing strong correlations between spatial reasoning and mathematics performance (e.g., Galton, 1880), and yet we know very little about what this means for educational application. Furthermore, Mathematics Education researchers who wish to glean insight and apply the knowledge created by Psychology and Cognitive Science must be keen discerners of not only what counts as "good science" but also must be able to find consensus in a body of literature where there often is none.

To summarize, we suggest that fruitful collaboration and transdisciplinary knowledge exchange is essential in addressing complex issues related to teaching and learning mathematics. Although Mathematics Education appears to both adopt and adapt ideas from other disciplines, it was rare to find examples of other disciplines borrowing from Mathematics Education. This is an unfortunate occurrence, as the knowledge of mathematics educators is critical in developing comprehensive accounts of how children (best) learn mathematics. It deserves to be restated that Mathematics Education researchers are ideally situated to facilitate the flow of information given their close contact with teachers, students, fellow math educators, and researchers from other disciplines. An important question Mathematics Education must ask itself is why other disciplines, such as Psychology and Cognitive Neuroscience, appear reluctant to borrow from Mathematics Education. Although answering this question is outside the purview of this article, further identification of connection gaps between disciplines can ultimately work toward bridging such gaps.

6 Conclusion

As already noted, consistent with its most common usages, we understand the term *transdisciplinary* to refer to a mode of shared inquiry in which experts from diverse domains gather around a complex problem of mutual concern. It is a research attitude that moves beyond the multidisciplinarian's tendency to combine insights and the interdisciplinarian's practice of conducting parallel analyses.

In our analysis, the literature reveals that spatial reasoning is a strong candidate for transdisciplinary inquiry—and yet, while it has been the focus of extensive multidisciplinary study and considerable interdisciplinary research, instances of genuine transdisciplinary research into spatial reasoning are virtually nonexistent. However, our main reason for raising the matter is not to provoke transdisciplinary studies of spatial reasoning. It is, rather, to raise the questions of, firstly, whether the time is right for the field of Mathematics Education to be more deliberately transdisciplinary in its work and, secondly, what pragmatic steps might be taken in that direction.

Clearly, our reply to the first of these queries is an emphatic "Yes." We get the same response whenever we pose that question to colleagues. Indeed, more often than not, the question is met with claims that the field is already transdisciplinary. After all, most phenomena of concern within Mathematics Education touch on issues that are of interest to researchers in a wide range of other disciplines. For example, significant theories and constructs around learning and knowing have been borrowed from Psychology, Anthropology, Mathematics, Philosophy, Biology, and Political Science. A list of examples might include constructs such as self-efficacy, situated learning, visualization, structured variation, embodied cognition, and alienation, to name a few.

However, as with spatial reasoning, while understandings of these and so many other phenomena have benefited from considerable peering over disciplinary fences, the phenomena themselves have rarely been the foci of genuine transdisciplinary study. And so, in response to the frequently encountered assertion that Mathematics Education is a transdisciplinary domain, we would respond that it certainly has the potential to serve as a transdisciplinary hub, but its strong multidisciplinarity and interdisciplinary heritage should not be confused with the complexities and entailments of transdisciplinary work. The call of transdisciplinary inquiry in not merely to *communicate*, but to *collaborate*.

We would argue that Mathematics Education is uniquely positioned to initiate and host transdisciplinary study of such phenomena as spatial reasoning. After all, the questions it addresses fulfill the requirement of complex problems that do not lend themselves to disciplinary or multidisciplinary solutions. Moreover, researchers in Mathematics Education must routinely bring the findings from other disciplines to pre-service and in-service professional training contexts in concrete and pragmatic terms. Rather than lamenting the dearth of transdisciplinary inquiry into the matters that are so close to us, it would seem to make more sense to expend those energies on inventing structures and protocols to enable transdisciplinary work.

To that end, we believe that our analysis and discussion offer some preliminary insight into how researchers might approach the prospect of collaborative inquiry. Nuanced understandings of who is already talking to whom about what afford not just useful entry points into already-established conversations, but offer useful hints for how we might format our own interests in ways that can be heard and engaged by experts in other disciplines. Phrased differently, and as illustrated by the two brief examples in the previous section, we believe that the onus is on us to serve as the connective tissue for Neuroscience, Psychology, Mathematics, and other bodies of researchers. This role entails careful study of discipline-specific vocabularies and methodologies, alongside the articulation of problems in ways that permit all potential research collaborators to identify and situate themselves. It might also require patience and perseverance. After all, the flexible structures, flat networks, and open-ended questions of transdisciplinary teams will not always be a comfortable fit in institutions that have been traditionally defined by rigid boundaries, vertical hierarchies, and fixed agendas. We would be remiss to end this discussion without flagging the extraordinary progress that has been made in recent decades in establishing meaningful and rich communications between Education and its "partner" disciplines. Consider, for example, that until relatively recently it was not unusual to hear Education described as an "Applied Psychology." An analysis of the educational research literature in the mid-twentieth century would bear out this assertion. That situation has changed dramatically. Today, few would take issue with the assertion that research in Genetics, Epigenetics, Neuroscience, Psychology, Sociology, Anthropology, and Ecology (and many other domains) has direct and immediate relevance to Education—and, indeed, our network analysis bears out this assertion. While the links between Education and these domains may not always be strong, and even less often two-way, they are nonetheless present. Education is an undeniably multidisciplinary discipline.

Our intention in this writing was to press this evolution further, in the invitation to consider the possibilities of transdisciplinarity as a core attitude across educational research endeavors. Ultimately, what we are calling for here is a reconsideration of how findings from other disciplines are either ignored, or translated, and reduced in classroom settings. With regard to our specific research interest, we advocate an appropriate transformation of spatial reasoning findings in other research literatures into spatially effective ways of thinking across swaths of curricular content. With regard to the broader research enterprise, we advocate an issue-specific, problem-based attitude toward inquiry—one that not only invites but also compels researchers from different domains to come together around matters of shared interest.

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